***This is a federally approved curriculum by the College Board and is used as a model and implemented in our A.P. course. There are no official CCSS's associated with Calculus, but it is certainly an extension of our high school curriculum.**

Kalkaska High School

AP® Calculus AB

Syllabus 2

Syllabus Number: 876074v1

Course Outline

By successfully completing this course, you will be able to:

- Work with functions represented in a variety of ways and understand the connections among these representations.
- Understand the meaning of the derivative in terms of a rate of change and local linear approximation, and use derivatives to solve a variety of problems.
- Understand the relationship between the derivative and the definite integral.
- Communicate mathematics both orally and in well-written sentences to explain solutions to problems.
- Model a written description of a physical situation with a function, a differential equation, or an integral.
- Use technology to help solve problems, experiment, interpret results, and verify conclusions.
- Determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Technology Requirement

I will use a Texas Instruments 84 Plus graphing calculator in class regularly. You will want to have a graphing calculator as well. I recommend the TI-84 and the TI-89. I have a classroom set of TI-84 Plus calculators, and some are available for extended checkout from the media center.

We will use the calculator in a variety of ways including:

- Conduct explorations.
- Graph functions within arbitrary windows.
- Solve equations numerically.
- Analyze and interpret results.
- Justify and explain results of graphs and equations. [C5]

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

A Balanced Approach

Current mathematical education emphasizes a "Rule of Four." There are a variety of ways to approach and solve problems. The four branches of the problem-solving tree of mathematics are:

- Numerical analysis (where data points are known, but not an equation)
- Graphical analysis (where a graph is known, but again, not an equation)
- Analytic/algebraic analysis (traditional equation and variable manipulation)
- Verbal/written methods of representing problems (classic story problems as well as written justification of one's thinking in solving a problem—such as on our state assessment) **[C3]**

> **C3**—The course provides students with the opportunity to work with functions represented in a variety of ways—graphically, numerically, analytically, and verbally—and emphasizes the connections among these representations.

Below is an outline of topics along with a tentative timeline. Assessments are given at the end of each unit as well as intermittently during each unit. Semester finals are also given.

Unit 1: Limits and Continuity (3–4 weeks) [C2]

> **C2**—The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; Integrals; and Polynomial Approximations and Series as delineated in the Calculus BC Topic Outline in the *AP Calculus Course Description*.

A. Rates of Change
   1. Average Speed
   2. Instantaneous Speed

B. Limits at a Point
   1. 1-sided Limits
   2. 2-sided Limits
   3. Sandwich Theorem

C. Limits involving infinity
   1. Asymptotic behavior (horizontal and vertical)
   2. End behavior models
   3. Properties of limits (algebraic analysis)
   4. Visualizing limits (graphic analysis)

D. Continuity
   1. Continuity at a point
   2. Continuous functions
   3. Discontinuous functions
      a. Removable discontinuity (0/0 form)

b. Jump discontinuity (We look at y = int(x).)
c. Infinite discontinuity

E. Rates of Change and Tangent Lines
1. Average rate of change
2. Tangent line to a curve
3. Slope of a curve (algebraically and graphically)
4. Normal line to a curve (algebraically and graphically)
5. Instantaneous rate of change

Unit 2: The Derivative (5–6 weeks) [C2]

A. Derivative of a Function
1. Definition of the derivative (difference quotient)
2. Derivative at a Point
3. Relationships between the graphs of f and f'
4. Graphing a derivative from data

**A CBL experiment is conducted with students tossing a large ball into the air. Students graph the height of the ball versus the time the ball is in the air. The calculator is used to find a quadratic equation to model the motion of the ball over time. Average velocities are calculated over different time intervals and students are asked to approximate instantaneous velocity. The tabular data and the regression equation are both used in these calculations. These velocities are graphed versus time on the same graph as the height versus time graph. [C3] [C5]

5. One-sided derivatives

B. Differentiability
1. Cases where f'(x) might fail to exist
2. Local linearity

**An exploration is conducted with the calculator in table groups. Students graph y1 = absolute value of (x) + 1 and y2 = sqrt (x^2 + 0.0001) + 0.99. They investigate the graphs near x = 0 by zooming in repeatedly. The students discuss the local linearity of each graph and whether each function appears to be differentiable at x = 0. [C4] [C5]

3. Derivatives on the calculator (Numerical derivatives using NDERIV)
4. Symmetric difference quotient
5. Relationship between differentiability and continuity
6. Intermediate Value Theorem for Derivatives

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C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally, and in written sentences.

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

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C. Rules for Differentiation
   1. Constant, Power, Sum, Difference, Product, Quotient Rules
   2. Higher order derivatives

D. Applications of the Derivative
   1. Position, velocity, acceleration, and jerk
   2. Particle motion

E. Derivatives of trigonometric functions

F. Chain Rule

G. Implicit Differentiation
   1. Differential method
   2. y’ method

H. Derivatives of inverse trigonometric functions

I. Derivatives of Exponential and Logarithmic Functions

Unit 3: Applications of the Derivative (5–6 weeks) [C2]

A. Extreme Values
   1. Relative Extrema
   2. Absolute Extrema
   3. Extreme Value Theorem
   4. Definition of a critical point

B. Implications of the Derivative
   1. Rolle’s Theorem
   2. Mean Value Theorem
   3. Increasing and decreasing functions

C. Connecting f’ and f’’ with the graph of f(x)
   1. First derivative test for relative max/min

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2. Second derivative
    a. Concavity
    b. Inflection points
    c. Second derivative test for relative max/min

* A matching game is played with laminated cards that represent functions in four ways: a graph of the function; a graph of the derivative of the function; a written description of the function; and a written description of the derivative of the function. **[C3]**

D. Optimization problems

E. Linearization models
  1. Local linearization

An exploration using the graphing calculator is conducted in table groups where students graph f(x) = (x^2 + 0.0001)^0.25 + 0.9 around x = 0. Students algebraically find the equation of the line tangent to f(x) at x = 0. Students then repeatedly zoom in on the graph of f(x) at x = 0. Students are then asked to approximate f(0.1) using the tangent line and then calculate f(0.1) using the calculator. This is repeated for the same function, but different x values further and further away from x = 0. Students then individually write about and then discuss with their tablemates the use of the tangent line in approximating the value of the function near (and not so near) x = 0. **[C3] [C4] [C5]

  2. Tangent line approximation
  3. Differentials

F. Related Rates

Unit 4: The Definite Integral (3–4 weeks) [C2]

A. Approximating areas
  1. Riemann sums
    a. Left sums
    b. Right sums
    c. Midpoint sums
    d. Trapezoidal sums

Here students are asked to input a program that will calculate trapezoidal sums for trapezoids of equal width. They are given this program. They are encouraged to think about altering it to be able to calculate rectangular sums as well. **[C5]

  2. Definite integrals

Students are asked to graph, by hand, a constant function of their choosing. Then they are asked to calculate a definite integral from x = -3 to x = 5 using known geometric methods. Students then share their work with their tablemates and are asked to come up with a table observation. Those observations are shared with other tables and a formula is discovered. **[C3]

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B. Properties of Definite Integrals
   1. Power rule
   2. Mean value theorem for definite integrals

**An exploration is conducted to show students the geometry of the mean value theorem for definite integrals and how it is connected to the algebra of the theorem. [C3]

C. The Fundamental Theorem of Calculus
   1. Part 1
   2. Part 2

Unit 5: Differential Equations and Mathematical Modeling (4 weeks) [C2]

A. Slope Fields

B. Antiderivatives
   1. Indefinite integrals
   2. Power formulas
   3. Trigonometric formulas
   4. Exponential and Logarithmic formulas

C. Separable Differential Equations
   1. Growth and decay
   2. Slope fields (Resources from the AP Calculus website are liberally used.)
   3. General differential equations
   4. Newton's law of cooling

D. Logistic Growth

Unit 6: Applications of Definite Integrals (3 weeks) [C2]

A. Integral as net change
   1. Calculating distance traveled (particle motion)
   2. Consumption over time
   3. Net change from data

B. Area between curves
   1. Area between a curve and an axis
      a. Integrating with respect to x
      b. Integrating with respect to y
   2. Area between intersecting curves
      a. Integrating with respect to x
      b. Integrating with respect to y

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C. Calculating volume
   1. Cross sections
   2. Disc method
   3. Shell method

Unit 7: Review/Test Preparation (time varies, generally 3–5 weeks)

A. Multiple-choice practice (Items from past exams—1997, 1998, and 2003 are used as well as items from review books I've purchased over the years.)
   1. Test taking strategies are emphasized
   2. Individual and group practice are both used

B. Free-response practice (Released items from the AP Central website are used liberally.)
   1. Rubrics are reviewed so students see the need for complete answers
   2. Students collaborate to formulate team responses
   3. Individually written responses are crafted. Attention to full explanations is emphasized **[C4]**

> **C4**—The course teaches students how to communicate mathematics and explain solutions to problems both verbally, and in written sentences.

Unit 8: After the exam…

A. Projects designed to incorporate this year's learning in applied ways

B. Research projects on the historical development of mathematics with a focus on calculus

C. Advanced integration techniques

D. A look at college math requirements and expectations including placement exams

Textbook:

Finney, Demana, Waits and Kennedy. *Calculus—Graphical, Numerical, Algebraic*. Third edition. Pearson, Prentice Hall, 2007.

This textbook will be our primary resource. You will benefit from reading it. It contains a number of interesting explorations that we will conduct with the goal that you discover fundamental calculus concepts. I will also explain topics in a way that students have found helpful over the years. I encourage cooperative learning, and I believe our entire class benefits from us all working together to help one another construct understanding. **[C4]**

My hope is that you want to learn as much as you can about calculus. Mathematicians have been responsible for many great developments throughout history. Much of our understanding of the universe is a direct result of the contributions of mathematicians. Who knows, perhaps we'll discover something during our course of studies. Whatever happens, I hope you learn to view math as more than just numbers, variables, processes, and algorithms. I hope you learn to apply your mathematical understanding to help you create a better understanding of the mathematical nature of our lives.